Communications Research Centre

AGILE FREQUENCY SELECTION FOR AN ANTENNA WITH FREQUENCY DEPENDENT BEAM FIRING ANGLE

by

H.C. Chan

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(Radar and Communications Technology Branch)

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TABLE OF CONTENTS

		PAGE
	ABSTRACT	1
1.	INTRODUCTION	1
2.	GEOMETRY OF COMBINED MECHANICAL AND FREQUENCY SCANNING	2
3.	THE RANDOM SEQUENCE	5
4.	AZIMUTHAL COVERAGE WITH FREQUENCY AGILITY	8
5.	CONSTRAINED FREQUENCY PERTURBATION FOR IMPROVED AZIMUTHAL COVERAGE	10
6.	PERIODICITY	23
7.	CONCLUSION	24
8.	ACKNOWLEDGEMENTS	24
9.	REFERENCES	24

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Abstract

(U) This work addresses the potential problems and their solution of employing a frequency squinting antenna in conjunction with frequency agility in a naval surveillance radar system. Frequency agility is an important feature in countering enemy jamming. However, when it is applied with a frequency squinting antenna, random beam firing angle will result. This random beam firing angle causes non-uniform azimuthal coverage. Consequently detection performance in certain azimuths scanned by the antenna will be degraded. An optimization technique is employed to obtain the sequence of agile frequency selections which provides a relatively uniform azimuthal coverage, while preserving the basic random properties of the nominal sequence of agile frequencies.

1. INTRODUCTION

(U) In modern radar systems frequency agility is an important feature to counter enemy jamming. The radar frequency is changed from pulse-to-pulse, or from burst-to-burst, so that an enemy cannot use spot frequency jamming but is forced to use less effective wideband barrage noise jamming or deploy sophisticated repeater jammers with a frequency following capability. With pulse-to-pulse frequency agility, the selection of the instantaneous radar frequency is random so that the enemy cannot predict with certainty what the frequency of the next pulse will be. The frequency of occurrence of the radar frequencies is roughly uniform so that one part of the radar band is not favoured over another.

(U) With a conventional (nonfrequency squinting) antenna the selection of random frequencies is straightforward, since the look direction is independent of frequency. This is not the case for frequency squinting antennas, however, because the frequency dependence of the beam direction can cause non-uniform azimuthal coverage and serious degradation in detection performance.

2. GEOMETRY OF COMBINED MECHANICAL AND FREQUENCY SCANNING

(U) Figure 1 depicts the geometry of the frequency squinting antenna beam firing direction. To facilitate the analysis of this problem, the following symbols representing the relevant parameters are defined:

ϕ = instantaneous beam firing angle with respect to ship heading, in degrees
Θ = bearing of the antenna normal with respect to ship heading, in degrees
α = azimuthal bias angle for the frequency squinting antenna, in degrees

Note: This is the beam pointing angle with respect to the antenna aperture normal at the lower limit of the agile frequency band. For travelling wave antennas, boresight beam condition is normally avoided because of the existence of resonance.

ω = antenna rotational rate in degree/sec
Ω = angular velocity of the antenna normalized to the radar pulse repetition interval (PRI) in degrees/PRIβ = frequency squinting sensitivity in degrees/GHz
f = instantaneous frequency, in GHz
f_1 = lower limit of the agile frequency band, in GHz
f_2 = upper limit of the agile frequency band, in GHz
PRF = radar pulse repetition frequency
γ = 3 dB one=way beamwidth, in degrees of the frequency squinting antenna at f_1

(U) A comparative simulation of the azimuthal coverage between a conventional antenna and a hypothetical frequency squinting antenna is performed. Except for the frequency squint, both antennas are assumed to have identical characteristics.

(U) At time t = 0 the normal to the antenna aperture coincides with the ship heading. Let T be the time interval between pulses (PRI), then

$$T = 1/PRF \tag{1}$$

(U) A pulse will be emitted whenever

$$t = nT \qquad n=0,1,2,... \tag{2}$$

(U) The index n will be called the pulse index. Since the angular velocity of the antenna is normalized to the system PRI, the angular position of the antenna beam at the time of pulse emission becomes a function of the pulse index n. It is assumed that there is an integer number of pulses emitted in each antenna revolution. The beam pointing direction of a conventional antenna is independent of radar frequency, hence it is governed only by the antenna rotational movement.

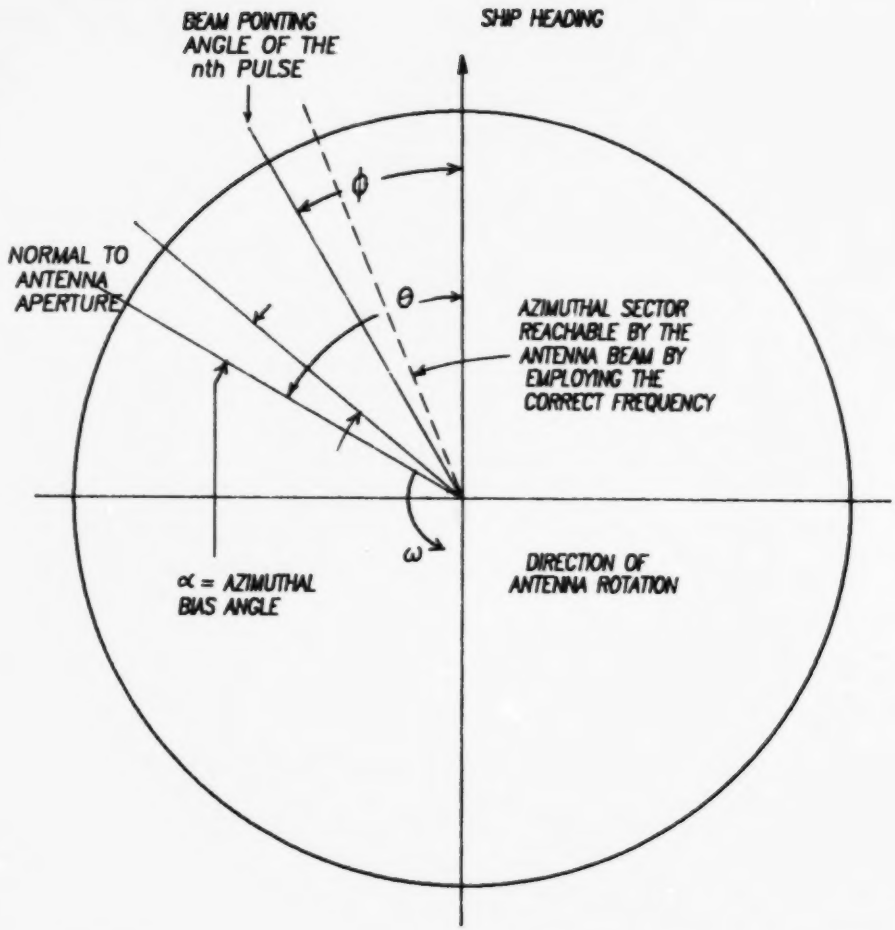

BEAM POINTING ANGLE OF THE nth PULSE
SHIP HEADING
ϕ
NORMAL TO ANTENNA APERTURE
θ
AZIMUTHAL SECTOR REACHABLE BY THE ANTENNA BEAM BY EMPLOYING THE CORRECT FREQUENCY
ω
DIRECTION OF ANTENNA ROTATION
α = AZIMUTHAL BIAS ANGLE

FIGURE 1. GEOMETRY OF THE FREQUENCY SQUINTING ANTENNA BEAM POINTING DIRECTION.

$$\phi_c(n) = \Theta(n) = \Omega n \qquad (3)$$

where $\phi_c(n)$ denotes the angle of the antenna beam with respect to the ship heading at the time the nth pulse is emitted. Subscript c denotes a conventional, mechanically rotated antenna.

(U) For the frequency squinting antenna, the beam pointing direction is a function of the frequency and is given by:

$$\phi_s(n,f) = \Theta(n)-[\alpha + \beta(f - f_1)]$$

$$= \Omega n-[\alpha + \beta(f - f_1)] \qquad (4)$$

where α, β, Ω and f_1 are defined at the beginning of this section.

$\phi_s(n,f)$ denotes the angle between the antenna beam and the ship's heading at the time the nth pulse of frequence f is emitted. Subscript s denotes a mechanically rotated, frequency squinting antenna.

The beamwidth of both antennas will be affected somewhat by the radar frequency. It is assumed that the beamwidth is a linear function of frequency is given by:

$$\gamma(f) = \gamma(f_1) - \delta(f - f_1) \qquad (5)$$

where $\gamma(f)$ is the 3 dB one-way beamwidth at frequency f and δ = slope of the change of beamwidth with respect to a change of frequency (degree/GHz)

This has been observed experimentally [1] to be a good approximation for the sandwich wire antenna being considered.

(U) For the purpose of the simulation, the following set of typical values is assigned to the antenna parameters

Antenna rotational rate ω = 120°/sec
PRF = 740 Hz
Frequency squinting sensitivity β = 24°/GHz
f_1 = 2.7 GHz
f_2 = 3.2 GHz
γ = 2.5° @ 2.7 GHz
δ = 0.8°/GHz

(U) Comparisons are made through the use of histograms of illumination. The histogram of illumination is a record of the number of pulses directed to each azimuth in one antenna revolution. The azimuth is divided into a number of equal sectors. For each pulse transmitted, one or more of the sectors will be illuminated by the 3 dB beamwidth. Exactly which sectors are illuminated will depend on the beam firing direction and the beamwidth at the chosen frequency. An acumulator is assigned to each of the azimuth sectors which is initially zeroed. At time t = 0, the antenna normal coincides with the ship heading. The first pulse is emitted at this time. Depending on the 3 dB beamwidth of this pulse a number of azimuthal sectors in the vicinity of the beam direction will be illuminated. The accumulator for each wholly illuminated sectors will be incremented by one. For azimuthal sectors which are only partially illuminated, the fraction corresponding to the illuminated part of the whole sector will be added to the accumulator.

3. THE RANDOM SEQUENCE

(U) The agile frequency is drawn from a sequence of discrete random numbers which assume one of N_f = 16 values (0-15). Each value will correspond to one of 16 frequencies equally spaced across the agile frequency band. [The restriction of the frequency division to discrete values of N_f is justified because the radar signal has a finite bandwidth]. The separation between the frequencies must be greater than the signal bandwidth to be significant. If the number, N_f is chosen to be a power of 2, one can utilize the so-called pseudo-noise (PN) binary sequence [2]. For example, the grouping of continuous binary digits in twos will provide 4 values: 00=0, 01=1, 10=2 and 11=3. Grouping in fours will provide 16 values.

(U) Figure 2 shows the schematic diagram of a 15-stage PN sequence generator. This PN sequence generator will produce a unique binary sequence of length $2^{15}-1 = 32767$.

(U) Figure 3 shows the histogram representing the relative frequency of occurrence of each of the 16 discrete values over one period of the PN sequence. Each random number is formed by four contiguous binary digits. Figure 4 shows the auto-correlation function of the random numbers. The auto-correlation function is defined here as:

$$a(k) = \sum_{n=1}^{N} [x(n)-\bar{x}][x(n+k)-\bar{x}] \qquad (6)$$

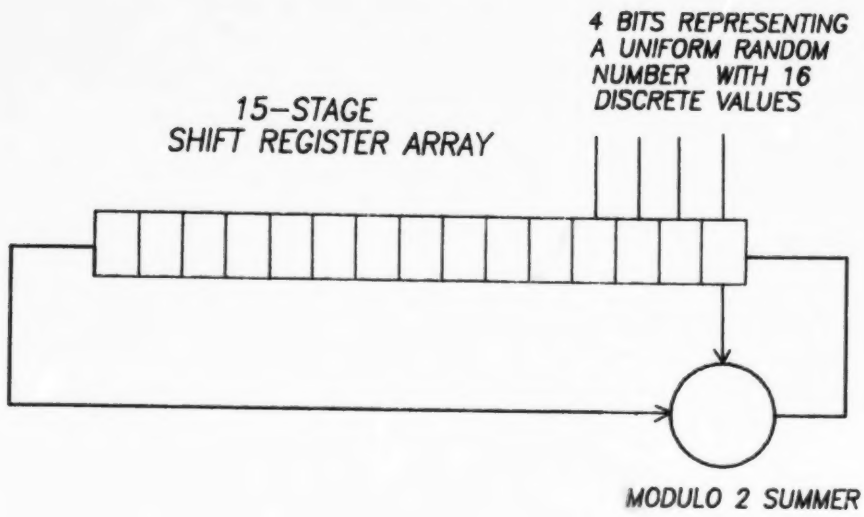


FIGURE 2. A 15–STAGE PSEUDO–NOISE GENERATOR.

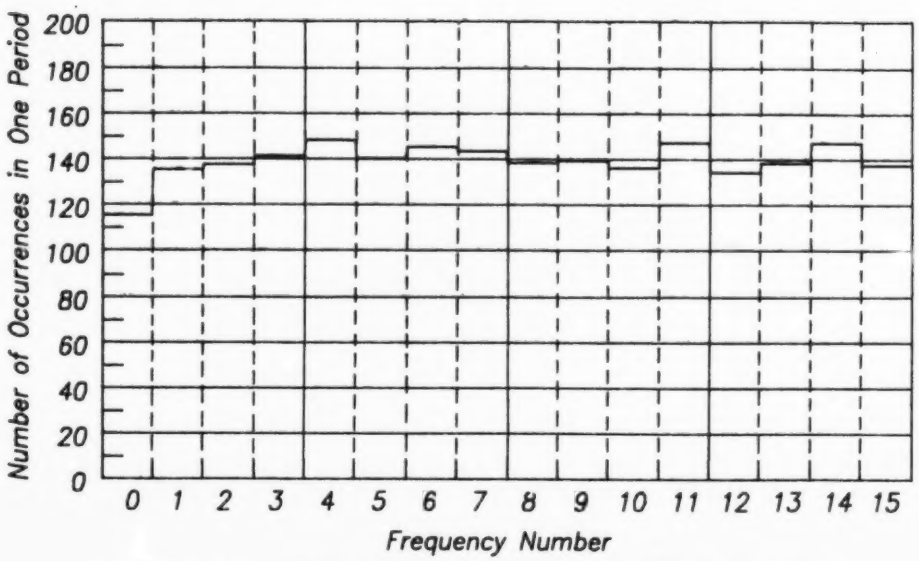


FIGURE 3. HISTOGRAM OF THE UNIFORM RANDOM SEQUENCE.

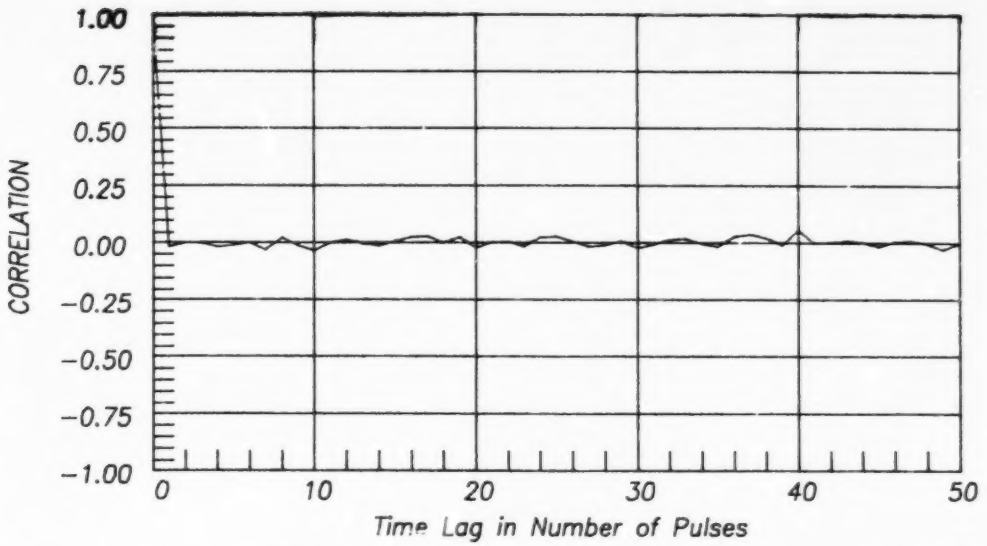


FIGURE 4. AUTOCORRELATION FUNCTION OF UNIFORM RANDOM SEQUENCE

where $\bar{x}$ = mean value of the random variable x

$$\bar{x} = \frac{1}{N_f} \sum_{i=0}^{N_f-1} i = 7.5 \qquad (7)$$

$$N_f = 16$$

assuming that x(n) are uniformly distributed over the 16 values from 0 to 15.

(U) It can be seen that the random numbers are sufficiently uniformly distributed, and they are essentially uncorrelated over a large number of samples. This ensures that it would be difficult for an enemy to predict the frequency of the next pulse, or to detect any bias in the frequency selection.

4. AZIMUTHAL COVERAGE WITH FREQUENCY AGILITY

(U) For a conventional reflector type antenna operating without frequency agility, the number of pulses on target per scan is simply given by:

$$\text{Hits per 3 dB beamwidth} = \frac{\Theta_{3dB} \times PRF}{\omega} = \frac{2.3 \times 740}{120} = 14.183 \quad (8)$$

where Θ_{3dB} = 3 dB beamwidth of antenna at mid agile frequency band

ω = antenna rotation rate in degrees/sec.

(U) Figure 5 shows the resulting histogram of illumination of a sector of 30° over one antenna revolution by the squinting antenna, when the above random sequence is used to select the agile frequency. The term histogram is used for lack of a more descriptive term. It should be emphasized that it is not a probabilistic parameter. It only gives a measure of how many pulses are to be received in any particular azimuth over one revolution. This histogram is typical of all azimuthal sectors. It shows that the azimuthal coverage is no longer uniform as in the case of the reflector type

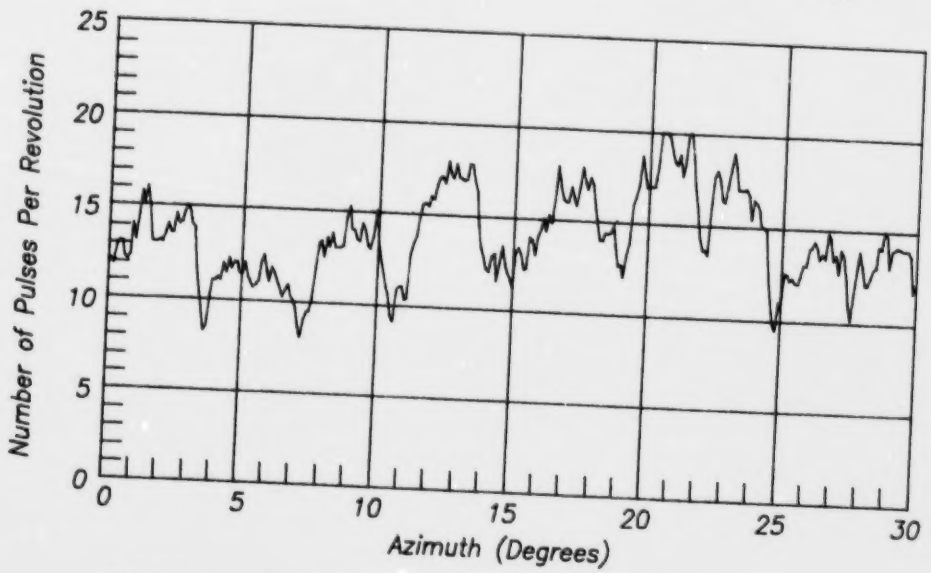


FIGURE 5. TYPICAL HISTOGRAM OF ILLUMINATION OF THE SQUINTING ANTENNA WITHOUT AGILE FREQUENCE SELECTION OPTIMIZATION.

antenna. (It should be noted that the use of this PN code is for simulation purpose only. In a practical system it would be prudent to employ more secure PN codes that would be difficult for the enemy to break under tactical conditions). Certain azimuths receive as many as 20 pulses, while others receive as few as 8 pulses over one revolution.

(U) For adequate detection performance, integration of pulses is required. In the case of pulse-to-pulse frequency agility, signal integration is necessarily noncoherent. The integration gain is dependent on the number of pulses available for integration. Figure 6 [3] shows the required signal-to-noise ratio (SNR) for 0.95 probability of detection (P_D) as a function of number of pulses integrated by a linear detector. These curves

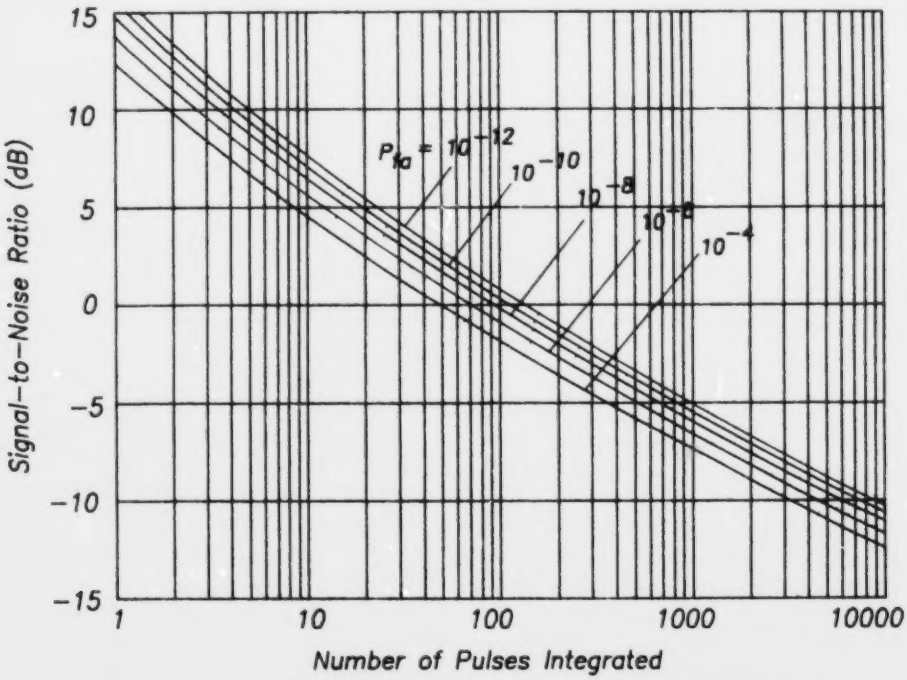


FIGURE 6. REQUIRED SNR AS A FUNCTION OF THE NUMBER OF NON−COHERENT PULSES INTEGRATED, FOR A LINEAR DETECTOR, 0.95 PROBABILITY OF DETECTION, NON−FLUCTUATING TARGETS, FOR FIVE VALUES OF P_{fa}.

are for nonfluctuating targets, and for five values of false alarm probability (P_{fa}). Assuming for the moment that a digital sample sorter is available to rearrange the returned signal in the proper azimuthal order, the SNR for each azimuth will depend on the number of pulses received. There is a difference of close to 3 dBs in the required SNR between the case of 20 pulses integrated and the case of 8 pulses, for the same P_D and P_{fa}.

Taking into account the fact that there is a difference of 3 dBs in the energy received by the target if the main beam is exactly on target compared to the case when the edge of the 3 dB-beamwidth frequeny is on target, there will be a substantial difference in the detection performance from one azimuth to another. It is apparent that some effort must be devoted to finding the proper frequency selection algorithm which can satisfy the frequency randomness requirement and, at the same time, provides relatively uniform azimuthal coverage.

(U) There is no unique solution to this problem. One possible approach is to use optimization techniques which is the topic of the next section.

5. CONSTRAINED FREQUENCY PERTURBATION FOR IMPROVED AZIMUTHAL COVERAGE

(U) In order to obtain a uniform coverage when the squinting antenna is used with frequency agility, the frequency selection must be modified. There are two potentially conflicting requirements. The first is the requirement of uniformity of frequency selection. The second is the uniformity of azimuthal coverage. A compromise must be struck between these two requirements. In principle one can start out with a sequence meeting one of the two requirements and then perturb the frequency to satisfy the other. It is highly likely that an attempt to improve one aspect will degrade the other but by observing certain constraints in the frequency perturbation, the two requirements can be met satisfactorily. The selection of radar frequency affects the distribution of frequencies as well as their correlation. These two measures, however, are ensemble averages; that is, the distribution and correlation of frequencies can be determined only after the entire sequence is determined. Consequently, it is more logical to start with a sequence of frequency selections which satisfy the distribution and correlation requirements and work to satisfy the uniform azimuthal coverage requirement by frequency perturbation.

(U) It is important to identify the objective and the constraints. Since the antenna rotational rate is ω and the pulse repetition frequency is PRF, the total number of pulses to be emitted per antenna revolution is:

$$N_p = PRF \times 360/\omega \tag{9}$$

(U) If the azimuthal coverage is divided into N_p sectors, then the angular extent of each sector will be:

$$\Delta\Theta = 360°/N_p \tag{10}$$

(U) The most uniform coverage would result if one and only one pulse is directed to the centre of each of the N_p azimuthal sectors.

Consequently the objective is to manipulate the frequency in order to achieve the above condition. This may be considered as a constrained optimization problem. One of the constraints to be observed is the resulting distribution of the frequencies selected. It must be reasonably uniform. If, during the course of modifying the frequency selection, certain frequencies are selected significantly more often than others, it would result in a bias in the frequency distribution. This could be exploited by enemy jammers. Another constraint is the correlation of the frequencies. There appears to be no direct control which can be applied to guarantee an adequately uncorrelated sequence. The undesirable effects can be minimized however by keeping the number of changes made to the original frequency selection to a minimum.

(U) Figure 7 is a tabulation of the number of pulses pointed to identical azimuthal sectors in one antenna revolution. Each sector is $\Delta\theta$ in angular extent. For this particular random sequence, the frequencies selected resulted in 792 azimuthal sectors to which no pulse is directed.

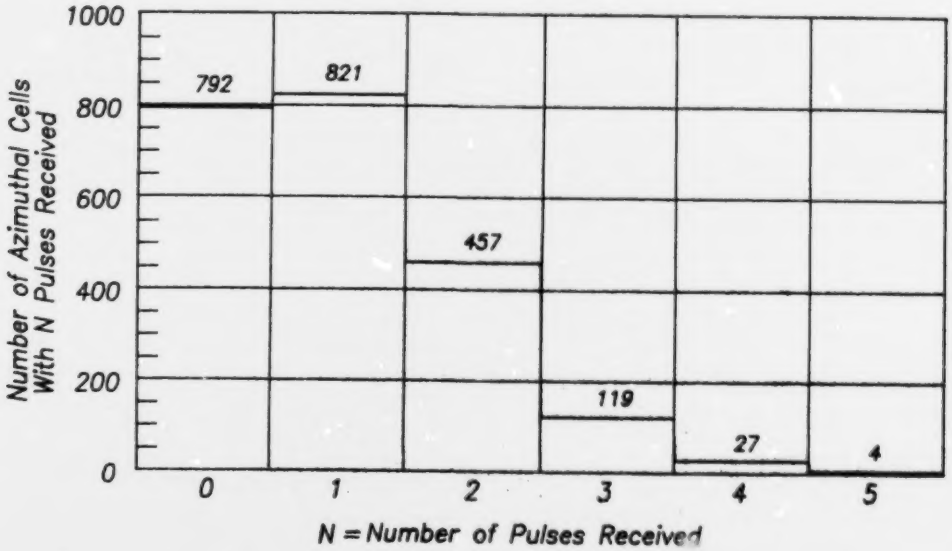


FIGURE 7. DISTRIBUTION OF MULTIPLE PULSE CELLS.

There are 821 azimuthal sectors which have exactly one pulse directed to them. There are 457, 119, 27 and 4 azimuthal sectors which received two, three, four and five pulses respectively during one antenna revolution. In subsequent discussion, in order to simplify the description, the term "empty

cell" will be used to describe the azimuthal sectors which did not receive any pulse in one antenna revolution. By "receiving", it is meant that the peak of the antenna beam falls into the azimuthal extent of that particular sector. The term "multiple pulse cell" will be used to describe the azimuthal sectors which receive more than one pulse during the course of one antenna revolution. The extra pulses of the multiple pulse cells can be diverted to neighbouring azimuthal sectors by a slight increment or decrement of the radar frequency. This can be accomplished without disturbing, to any significant degree, the inherent randomness of the original sequence. The redirection of a pulse to other azimuthal sectors by changing its frequency will be called "beam switching".

(U) The large fluctuation of azimuthal coverage is due to congregation of empty cells and multiple pulse cells. Table 1 tabulates the number of consecutive empty cells in one antenna revolution. If cells nearby have

(U) Table 1 Distribution of consecutive empty cells

No. of consecutive empty cells revolution	No. of occurrence per antenna
1	315
2	115
3	51
4	13
5	3
6	2
7	0
8	1

multiple pulses, these empty cells can be filled by diverting one of the extra pulses from the multiple pulse cells with a slight change in its frequency. However, there is a moderate number of empty cells which are joined together. For example, there is a string of eight azimuthal sectors which are not illuminated. Since the ability of the antenna to back scan is limited by the squinting range, it is highly likely that a number of the empty cells cannot be covered by directly switching the extra pulses in the multiple pulse cells.

(U) Assuming that the antenna is rotating in the counter-clockwise direction and that the ship heading is designated as the true north in the radar display, the beam firing angle will always be lagging behind the antenna normal. The converse is true if the antenna is rotating in the clockwise direction or the signal is being fed from the opposite end of the antenna. For the purpose of illustration, the convention depicted in Figure 1 will be used.

(U) Suppose in one antenna revolution, the jth azimuthal cell has received N pulses with frequencies f_1, f_2, f_3,..., f_N, respectively, then N-1 of these pulses can be diverted so that they can be pointed to empty cells. Suppose the (j-1)th cell is an empty cell, in order to direct one of the extra pulses to it, the frequency of that pulse must be changed by the amount

$$\Delta f = \frac{\Delta\theta}{\beta} \tag{11}$$

where β is the squinting sensitivity and
$\Delta\theta$ is defined in (10)

(U) If the angular distance between the multiple pulse cell and the empty cell is too great, then the frequency increment required may cause the agile frequency range to be exceeded. This difficulty can be alleviated by involving all the pulses directed to azimuthal cells located between the empty cell and the multple pulse cell. For instance, let the jth cell be a multiple pulse cell and the (j-k)th cell be an empty cell. There are k azimuthal cells separating the empty cell and the multiple pulse cell. The change in frequency required to direct an extra pulse from the jth cell to the (j-k)th cell is therefore:

$$\Delta f' = k\Delta f = \frac{k\Delta\theta}{\beta} \tag{12}$$

Suppose now that the frequency of the pulse in question can only be incremented by Δf without exceeding the agile frequency range. Consequently, it cannot be used to directly fill the empty cell; however, it can be used to point the beam to its immediate neighbor, the (j-1)th cell, using a frequency increment of Δf. The pulse originally assigned to the (j-1)th cell can then be directed to the (j-2)th cell by introducing the same amount of frequency increment. This process continues until the empty cell is filled.

(U) It is possible that even this procedure will fail to fill all the empty cells because of the frequency agility band limitation. Furthermore, this procedure, if applied indiscriminately, will destroy the uniformity property of the frequency distribution. Frequencies at the agile frequency band edges would tend to occur more often. Thus a better procedure is needed to complete the task.

(U) In the following, a set of steps is outlined which will result in a uniform histogram of illumination for one antenna revolution while, at the same time, attempting to maintain random frequency assignments.

a. A nominal frequency selection sequence is determined from a discrete uniform random number generator. The selection has N_p entries. The frequencies are indexed and stored in an array so that the kth entry represents the frequency used for the kth pulse.

b. A coarse histogram of illumination is constructed based on the nominal sequence by computing the exact direction of the beams. The circumference of the radar coverage is divided into N_p sectors, each with an azimuthal width given by (10). The cells are indexed in ascending order with the first cell representing the azimuthal cell in the ship's heading. There is an accumulator assigned to each of the azimuthal cell. If the beam pointing direction of a pulse falls in one of the cells, the accumulator assigned to this cell is incremented by one.

c. The frequencies of the pulses are then adjusted slightly so that the beam will be directed precisely at the centre of the cell.

d. A beam destination table is also constructed by recording the index of the azimuthal cell to which a pulse is directed.

e. The coarse histogram of illumination is scanned for multiple pulse cells. Once a multiple pulse cell, say the kth cell, is located, the cells immediately before and after the kth cell will be examined to determine if it is an empty cell. If it is, then one of the pulses originally assigned to the kth cell will be switched to it by changing its frequency by an amount of $\pm \Delta f$ given by (11). The plus sign is taken when the empty cell is lagging the kth cell, while the negative sign is taken if the empty cell is leading the kth cell in azimuth. The resulting frequency is checked to ensure that it does not exceed the agile frequency band limits. the steps are repeated until all empty cells have only single pulse neighbours. The beam destination table and the histogram of illumination are updated after each beam switching has taken place.

f. The resulting coarse histogram of illumination is again scanned for multiple pulse cells. When one is located, the closest empty cell is found. It is then filled by switching the pulse originally directed to its immediate neighbouring cell located between the empty cell and the multiple pulse cell. Thus the empty cell effectively propagates towards the multiple pulse cell. This continues until the empty cell appears immediately next to the multiple pulse cell which can then be filled by assigning one of the extra pulses to it. These steps are repeated until all empty cells are filled except those which cannot be

reached without employing frequencies exceeding the agile frequency band limits.

(U) Steps a to f will be called preliminary beam switching and they do not involve any optimization.

g. After the above steps are performed the histogram of illumination will be fairly uniform except for a small number of spots where empty cells cannot be eliminated by simple beam switching. This may be due to the large angular distance between the multiple pulse cell and the empty cell, or it may be due to the fact that the frequencies of some pulses are very close to the agile frequency band edge and cannot be incremented or decremented any further. In any case, the solution would involve switching beams selectively.

(U) At this point it would be useful to introduce an objective function which is a function of the frequency histogram. The frequency histogram is a tally of the number of occurrences of each of the N_f discrete frequencies over one antenna revolution. Because of the way the discrete random numbers are obtained, the nominal frequency histogram is fairly uniform except for the smallest frequency value. After preliminary beam switching, the uniformity of the frequency distribution will be degraded. The ensuing steps are designed to determine the proper switching of pulses so that the remaining empty cells can be eliminated while, as far as possible, the uniformity of the frequency histogram.

(U) The objective function is defined as:

$$F = \frac{1}{N_f} \sum_{i=1}^{N_f} (NF_i - F_M)^2 \qquad (13)$$

where NF_i is the number of occurences of the ith discrete frequency, and
F_M is the mean value of the frequency of occurrence

i.e., $F_M = N_p/N_f$

(U) The procedures outlined below may be considered locally optimal in the sense that they are concerned with finding the best way to switch a pulse originally directed to a particular multiple pulse cell to a particular empty cell. Consider a multiple pulse cell i and an empty cell ℓ.

Assume that there are K azimuthal cells separating them as depicted in Figure 8. Depending on the number of intermediate cells to be involved in the

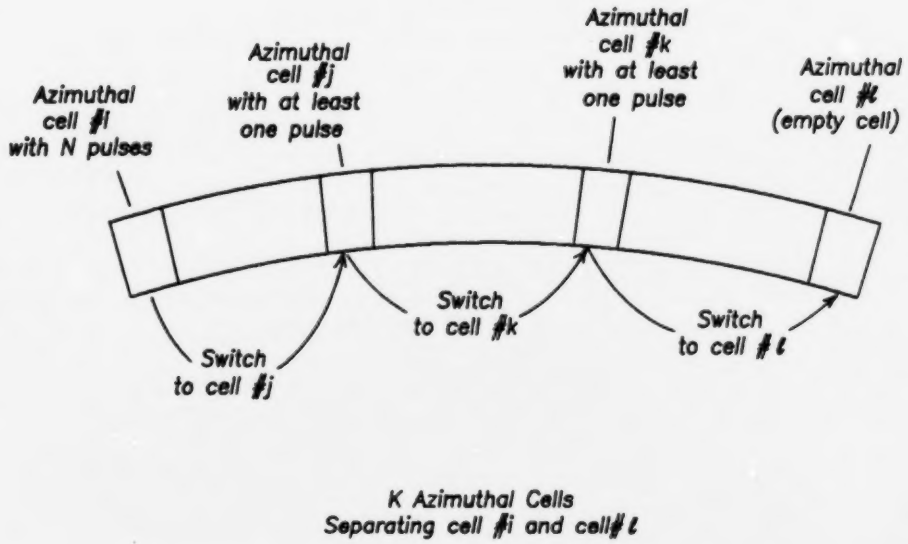

Azimuthal cell #l with N pulses
Azimuthal cell #j with at least one pulse
Azimuthal cell #k with at least one pulse
Azimuthal cell #ℓ (empty cell)
Switch to cell #j
Switch to cell #k
Switch to cell #ℓ
K Azimuthal Cells Separating cell #i and cell#ℓ

FIGURE 8. MULTIPLE-STAGE BEAM SWITCHING STRATEGY.

beam switching, there are a large number of alternative ways in which the empty cell can be filled. For instance, it may be possible to first divert one of the N pulses which are originally directed to the multiple cell i to the jth cell, and then change the frequency of the pulse for the jth cell so that it will point to another intermediate cell k. Finally the frequency of the pulse for cell k is changed so that it will now point to the empy cell ℓ. The criterion for selecting a particular way of beam switching is that it minimizes the objective function (12). This problem can be solved using dynamic programming [4] for multiple stage decision processes. This approach, however, requires a large amount of memory because it is a delayed decision process. All relevant items must be temporarily stored as alternative decisions are attempted at each stage. Alternatives which result in a higher objective function are purged, thereby reducing the total memory requirement. It is much more effective than direct enumeration which attempts all possible alternatives and then selects the optimum. For the problem at hand, the dynamic programming approach becomes very cumbersome

when the number of intermediate beam switches is large. If, on the other hand, the number of intermediate beam switches is limited to one, then the problem becomes a 2-stage decision process. In this case the dynamic programming approach is feasible and can be handled with direct enumeration. Aside from practical considerations, there are other reasons to limit the problem to a two-stage beam switching problem. The first is the fact that the majority of empty cells can be reached by pulses originally assigned to a multiple pulse cell via only one intermediate cell. The second is that in order to minimize the detrimental effects of beam switching on the random properties, the number of frequency changes should be kept to a minimum. Finally in the rare occasions that an empty cell cannot be filled with only one two-stage beam switch, then two or more two-stage beam switches can be cascaded to complete the task.

(U) An example should aid to illustrate the operations of these procedures. Assume that azimuthal cell i is a multiple pulse cell and cell ℓ is an empty cell. The problem is to find a way of switching one of the extra pulses originally directed to cell i to cell ℓ. There are many alternative ways in which this can be accomplished. Figure 9 symbolically shows just two of such alternatives.

(U) For alternative No. 1, the beam switching sequence is as follows:

i. The pulse originally directed to cell i is redirected to cell j.
ii. The pulse originally directed to cell j is redirected to cell ℓ.

(U) For alternative No. 2 the beam switching sequence is as follows:

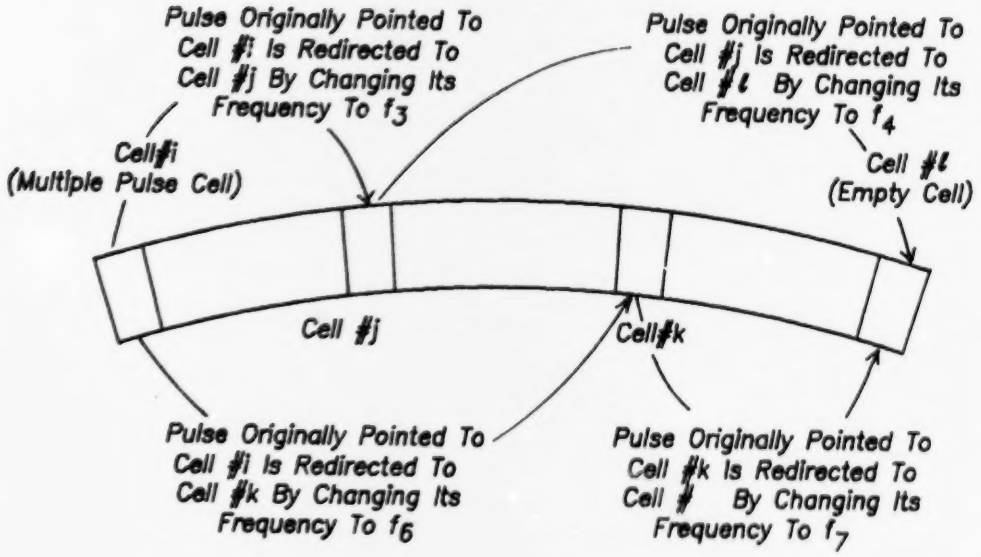
i. The pulse originally directed to cell i is redirected to cell k.
ii. The pulse originally directed to cell k is redirected to cell ℓ.

(U) The selection is based on the inflicted cost between choosing cell j or cell k as the intermediate cell.

(U) In addition the following items are given:

a. the frequency of the extra pulse originally pointed to cell i is f_1.
b. The frequency of the only pulse pointed to cell j is f_2.
c. If the pulse originally pointed at cell i is to be switched to cell j, its frequency must be changed to f_3.
d. If the pulse originally pointed to cell j is to be switched to cell ℓ, its frequency must be changed to f_4.
e. The frequency of the only pulse pointed at cell k is f_5.
f. If the pulse originally pointed at cell is to be switched to cell k, its frequency must be changed to f_6.
g. If the pulse originally pointed at cell k is to be switched to cell ℓ, its frequency must be changed to f_7.

Beam Switching Alternative No.1


Pulse Originally Pointed To Cell #i Is Redirected To Cell #j By Changing Its Frequency To f_3
Pulse Originally Pointed To Cell #j Is Redirected To Cell #ℓ By Changing Its Frequency To f_4
Cell #i (Multiple Pulse Cell)
Cell #ℓ (Empty Cell)
Cell #j
Cell #k
Pulse Originally Pointed To Cell #i Is Redirected To Cell #k By Changing Its Frequency To f_6
Pulse Originally Pointed To Cell #k Is Redirected To Cell # By Changing Its Frequency To f_7

Beam Switching Alternative No.2

The Costs For The Two Alternatives Are Computed, And The One With The Smaller Cost Is Selected

FIGURE 9. TWO–STAGE BEAM SWITCHING STRATEGY.

(U) If the first alternative is chosen, the count of the frequency histogram for f_1 will be reduced by one, and the count for f_3 will be increased by one. Similarly, the count for f_2 will be decreased by one, and the count for f_4 will be increased by one. The resulting objective function due to this sequence of beam switching is:

$$\begin{aligned} F_1 = \sum_{i\neq 1}^{N_f} (NF_i - F_M)^2 &+ (NF_1 - 1 - F_M)^2 \\ i\neq 2 \quad &+ (NF_3 + 1 - F_M)^2 \\ i\neq 3 \quad &+ (NF_2 - 1 - F_M)^2 \\ i\neq 4 \quad &+ (NF_4 + 1 - F_M)^2 \end{aligned} \tag{14}$$

(U) If the alternative No. 2 is chosen, the count for f_1 will be reduced by one and the count for f_6 will be increased by one. Similarly the count for f_5 will be decreased by one and the count for f_7 will be increased by one. The resulting objective function due to this sequence of beam switching is

$$\begin{aligned} F_2 = \sum_{i\neq 1}^{N_f} (NF_i - F_M)^2 &+ (NF_1 - 1 - F_M)^2 \\ i\neq 6 \quad &+ (NF_6 + 1 - F_M)^2 \\ i\neq 5 \quad &+ (NF_5 - 1 - F_M)^2 \\ i\neq 7 \quad &+ (NF_7 + 1 - F_M)^2 \end{aligned} \tag{15}$$

The choice between these two alternatives is determined by selecting the one which yields the smaller objective function F. The optimum policy is found by considering all possible intermediate cells such as cell j and k in the above example. The selected alternative will be replaced whenever another alternative yields a smaller objective function F. These optimization procedures continue until all empty cells in the coarse histogram of illumination are filled.

(U) A set of subroutines is written which implement these procedures together with a program which demonstrates their use.

(U) Figure 10 shows the detailed histogram of illumination resulting from a frequency selection obtained by this program. Trace A is the coverage before optimization. Trace B shows the azimuthal coverage after optimization. It can be seen that the resulting histogram of illumination is very uniform. The total number of pulses received at any azimuth varies within plus and minus one pulse.

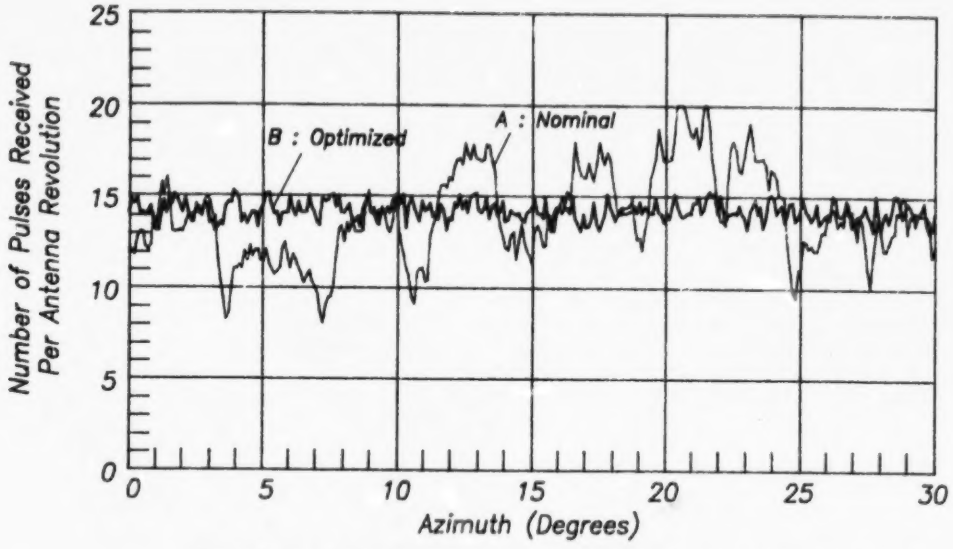


FIGURE 10. COMPARISON OF AZIMUTHAL COVERAGE OF AGILE FREQUENCY SELECTIONS WITH AND WITHOUT OPTIMIZATION.

(U) It remains to check the random properties of the optimized frequency selection. After optimization the frequencies obtained are quantized to the nearest of the 16 values available. Figure 11 is the frequency histogram of the optimized sequence. As can be seen it is fairly uniform. Figure 12 shows the autocorrelation function of the optimized frequency selection computed from the quantized frequencies. It shows no increase in correlation among frequencies.

(U) The optimized results for azimuthal coverage obtained above (Figure 10) assume that the radar frequency can be changed continuously. It would be of interest to find what the degradation would be in the azimuthal coverage if the radar frequency is quantized. Figure 13, Figure 14 and Figure 15 show the resulting histogram of illumination when the optimized frequency selection is quantized to 16,32 and 64 discrete levels respectively. It can be seen that at 64 levels, the azimuthal coverage is quite acceptable.

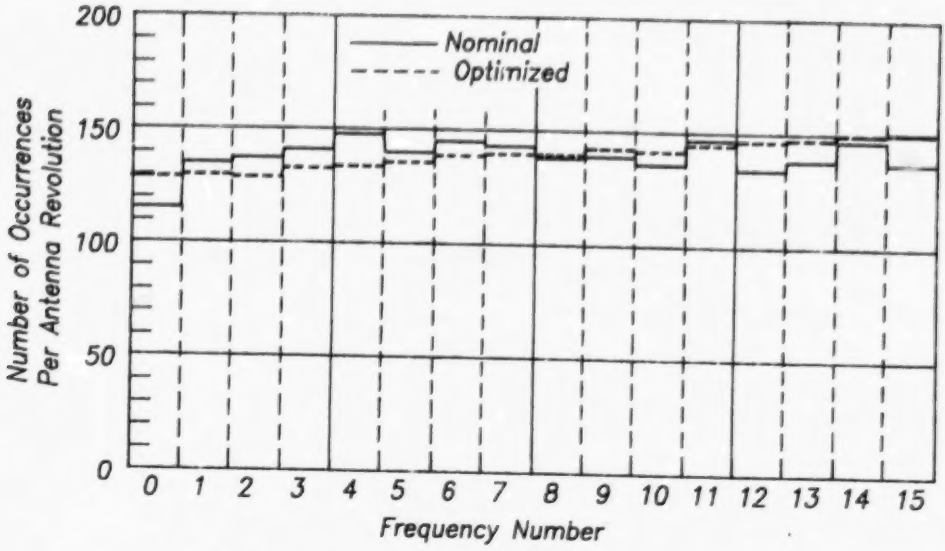


FIGURE 11. COMPARISON OF FREQUENCY HISTOGRAMS OF THE NOMINAL AND THE OPTIMIZED AGILE FREQUENCY SELECTIONS.

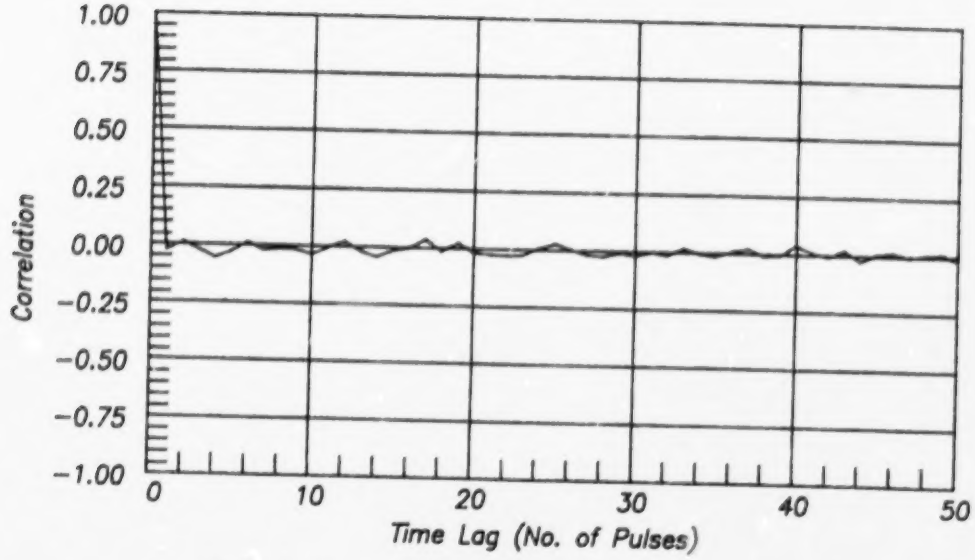


FIGURE 12. AUTOCORRELATION FUNCTION OF THE OPTIMIZED AGILE FREQUENCY SELECTION.

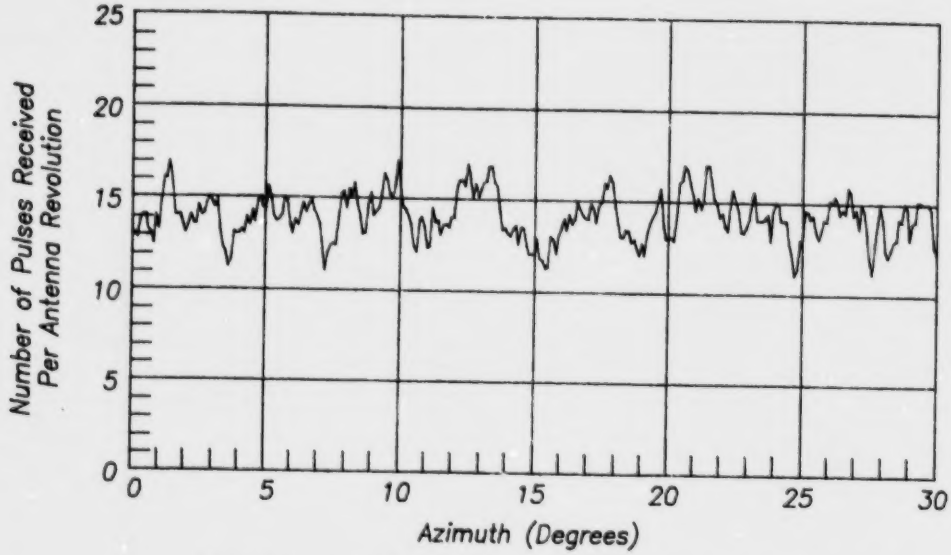


FIGURE 13. TYPICAL HISTOGRAM OF ILLUMINATION FOR THE OPTIMIZED AGILE FREQUENCY SELECTION QUANTIZED TO 16 DISCRETE LEVELS.

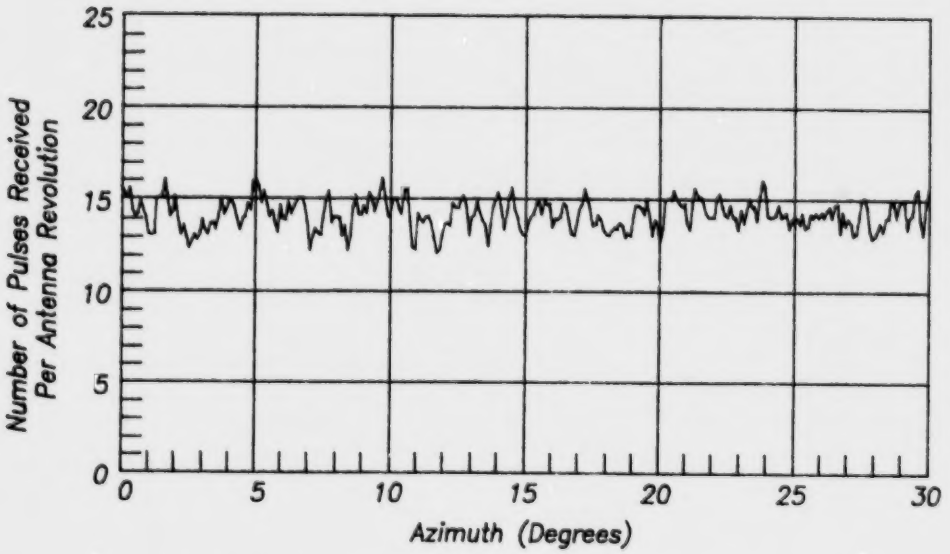


FIGURE 14. TYPICAL HISTOGRAM OF ILLUMINATION FOR THE OPTIMIZED AGILE FREQUENCY SELECTION QUANTIZED TO 32 DISCRETE LEVELS.

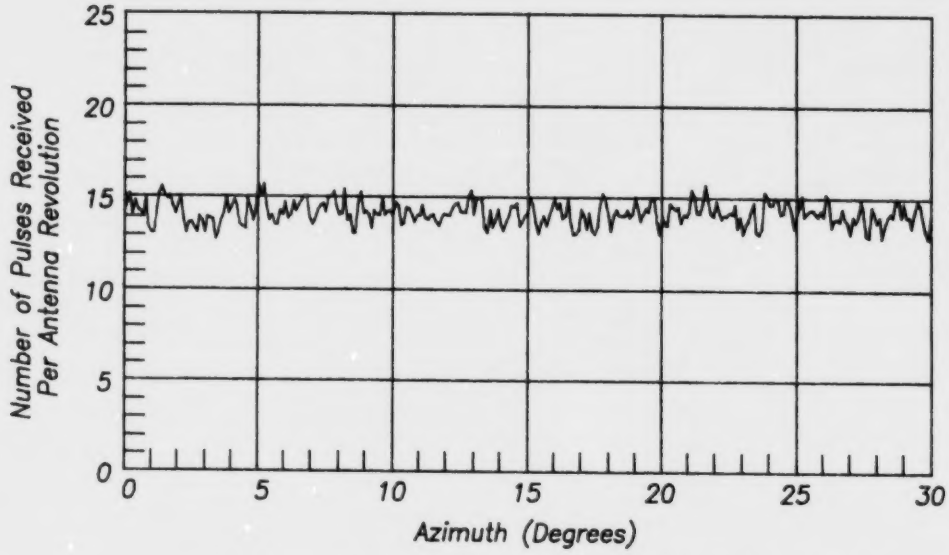


FIGURE 15. TYPICAL HISTOGRAM OF ILLUMINATION FOR THE OPTIMIZED AGILE FREQUENCY SELECTION QUANTIZED TO 64 DISCRETE LEVELS.

6. PERIODICITY

(U) The sequence of agile frequency selection obtained with the above procedures contains N_p entries, which is the total number of radar pulses emitted per antenna revolution. Thus by the time the antenna completes one revolution, the sequence would have been exhausted. The same sequence can be repeated and the uniformity in azimuthal coverage will be assured. However, this creates a periodicity in the frequency selection. If an enemy ESM receiver possesses some memory capability, it is conceivable that this periodicity can be exploited. This is not an insurmountable problem. One way to reduce the periodicity is to have several random sequences each of which will provide uniform azimuthal coverage. These sequences will be constrained such that they are identical in their entries at the beginning of the sequences. This will enable the sequences to be switched from one to the other without losing the uniformity of azimuthal coverage. The switching of the sequences can be done randomly, thereby making the apparent period extremely long. Nevertheless, this problem must be considered when evaluating the performance of a practical system.

7. CONCLUSION

(U) The use of frequency agility in a radar system with a mechanically rotated, frequency squinting, antenna causes a random beam pointing direction dependent on the radar frequency employed. This random beam pointing direction interacts with the mechanical rotational movement of the antenna and causes a non-uniform distribution of the transmitted pulses over all azimuths. As a result, detection performance may differ significantly from one azimuth to another. For a simple naval surveillance radar system, this problem can be alleviated by employing optimization techniques in the selection of the sequence of agile frequencies. Simulation results showed that, with the agile frequency selected from 32 or 64 discrete frequencies spaced equally across the agile frequency band, the azimuthal coverage can be brought to within ± 2 and ± 1 pulses, respectively, of that for the conventional antenna where the average number of pulses per beam is 14. The difference in detection performance, with one pulse more or less integrated, would have a neglible effect on performance. The same technique can be employed to obtain the agile frequency selection in the burst-to-burst mode when MTI filtering is used.

(U) Although the frequency squint does not seem to present an insurmountable problem, it does represent an added constraint on the signal processing system. Consequently, the effect of the frequency squint on the overall signal processing system performance must be analyzed when a specific system is implemented.

8. ACKNOWLEDGEMENTS

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This work addresses the potential problems and their solution of employing a frequency squinting antenna in conjunction with frequency agility in a naval surveillance radar system. Frequency agility is an important feature incountering enemy jamming. However, when it is applied with a frequency squinting antenna, random beam firing angle will result. This random beam firing angle causes non-uniform azimuthal coverage. Consequently detection performance in certain azimuths scanned by the antenna will be degraded. An optimization technique is employed to obtain the sequence of agile frequency selections which provides a relatively uniform azimuthal coverage, while preserving the basic random properties of the nominal sequence of agile frequencies.

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